

Distributed quantum computing on superconducting qutrits with dark microwave photons

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We present a one-step scheme to construct the controlled-phase gate deterministically on remote transmon qutrits trapped in different resonators connected by a superconducting transmission line for a distributed quantum computing. Different from previous work on remote superconducting qubits, the present gate is implemented with coherent evolutions of the entire system in the all-resonance regime assisted by the dark microwave photons to be robust against the transmission line loss, which allows the possibility of the complex designation of a long-length transmission line to link lots of circuit QEDs. This gate is a fast quantum entangling operation with a high fidelity of about 99%. Compare with previous works in other quantum systems for a distributed quantum computing, under the all-resonance regime, the present proposal does not require classical pulses and ancillary qubits, which relaxes the difficulty of its implementation largely.

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I. INTRODUCTION

Quantum computation (QC) [1], as an interdisciplinary research of computer science and quantum mechanics, has attracted much attention in recent years. It can implement the famous Shor's algorithm [2] for the factorization of an n -bit integer exponentially faster than the classical algorithms and the Grover's algorithm [3] or the optimal Long's algorithm [4] for unsorted database search. Various quantum systems have been used to implement QC, such as photons [5–9], nuclear magnetic resonance [10–12], diamond nitrogen-vacancy center [13–15], and cavity quantum electrodynamics (QED) [16]. Among the quantum systems, circuit QED [17, 18], composed of a superconducting qubit (SQ) coupled to a superconducting resonator (SR), provides a good platform for the implementation of QC because of its good ability of the large-scale integration and the accurate manipulation on the SQ [19].

Circuit QED has been studied a lot for achieving the basic tasks of QC on SQs or SRs, such as the construction of the single-qubit and the universal quantum gates [20–25], entangled state generation [26–30], and the measurement and the non-demolition detection on SQs or SRs [31–33]. The types for integrating the SQs and the SRs mainly contain some SQs coupled to a SR bus [34] or some SRs coupled to a SR bus [35] or a SQ [36–38]. At present, it is hard to integrate lots of SQs or SRs in a quantum-bus-based processor to achieve the complex universal QC. Further scaling up the number of SQs or SRs requires linking the distant circuit QED systems to form a quantum network [39–45] introduced by the *distributed quantum computing* [46], in which a quantum computer can be seen as a quantum network of distant local processors with only a few qubits and are connected by quantum transmission lines (TL). As the key problem in the realization of the distributed quantum computing, quantum entanglement and universal quantum gate on remote qubits have been discussed in some other systems [47–50]. For example, Cirac *et al.* [51] proposed a scheme to achieve the ideal quantum transmission between atoms trapped at spatially separated nodes in 1997. In 2004, Xiao *et al.* [52] realized the controlled phase (c-phase) gate between two rare-earth ions embedded in the respective microsphere cavities assisted by a single-photon pulse in sequence. In 2011, Lü *et al.* [53] proposed two schemes to complete the entanglement generation and quantum-state transfer between two spatially separated semiconductor quantum dot molecules.

To achieve the universal quantum gate on distant qubits trapped in different cavities connected by the TLs, realistic flying-photon qubit or adiabatic processes and the local operations are required. On one hand, there are some works which studied the quantum network by using the dark photon in the TL in other quantum systems. In 2007, Yin *et al.* [54] presented some schemes to achieve the state transfer and quantum entangling gates deterministically between the remote multiple two-level atoms trapped in different cavities connected by an optical fiber, in which the c-phase gate should be completed by using the “dipole blockade” effect among atoms in a cavity and it needs not to populate the realistic photons in the fiber. In 2014, Clader [55] presented an adiabatic scheme to transfer a microwave quantum state from one cavity to another, assisted by an optical fiber which is robust against both mechanical and fiber loss. On the other hand, one should transfer the microwave photon to the optical photons to link the remote SQs. In 2015, Yin *et al.* [56] proposed a scheme to achieve the quantum networking of SQs based on the optomechanical interface.

To implement the distributed quantum computing on remote SQs trapped in different SRs connected by a superconducting TL, one should overcome the decay of the TL as the more the complicated designation and a longer length for the TL is, the

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bigger the decay of the microwave photon in it becomes. In this paper, we propose a scheme for the construction of the c-phase gate on two remote transmon qutrits trapped in different SRs connected by a superconducting TL for the distributed quantum computing on SQs. Our scheme works in the all-resonance regime by letting the frequencies of the qutrits and the resonators equal to each other. The scheme can be achieved with just one step assisted by the dark microwave photons in the TL, without requiring classical pulses and ancillary qubits, which relaxes the difficulty of its implementation in experiment largely. Using the dark microwave photons in TL to reduce the requirement of the quality factor of the TL allows the complex designation of a long-length TL to link lots of remote circuit QEDs. The fidelity of the present c-phase gate is beyond 99% by using the numerical simulation with the feasible parameters.

II. BASIC THEORIES

Let us consider a distributed quantum computing composed of two remote superconducting qubits q_1 and q_2 trapped in two single-mode high-quality superconducting resonators r_a and r_b , respectively, which are connected by a superconducting TL r_f , shown in Fig. 1. The Hamiltonian of this device is (in the interaction picture with $\hbar = 1$)

$$\begin{aligned} H &= H_1^a + H_2^b + H_f^{a(b)} \\ &= g_1^a(a^\dagger \sigma_1^- e^{-i\delta_1^a t} + a \sigma_1^+ e^{i\delta_1^a t}) + g_2^b(b^\dagger \sigma_2^- e^{-i\delta_2^b t} + b \sigma_2^+ e^{i\delta_2^b t}) \\ &\quad + \sum_{j=1}^{\infty} g_{f,j}^l [f_j(a^\dagger + (-1)^j e^{i\phi} b^\dagger) + H.c.], \end{aligned} \quad (1)$$

where H_1^a , H_2^b , and $H_f^{a(b)}$ are the interaction Hamiltonians of the subsystems composed of q_1 and r_a , q_2 and r_b , and r_f and r_a (r_b), respectively. $H_f^{a(b)}$ applies to the high-finesse resonators and resonant operations over the time scale much longer than the TL's round-trip time [57]. $\delta_J^I = \omega_I - \omega_J$ ($I = a, b$ and $J = 1, 2, f$). ω_a , ω_b , and ω_f are the transition frequencies of resonators r_a , r_b , and the TL r_f , respectively. ω_1 and ω_2 are the transition frequencies of the qubits q_1 and q_2 , respectively. a^\dagger , b^\dagger , and f^\dagger are the creation operators of the resonators r_a , r_b , and the TL r_f , respectively. σ_1^+ and σ_2^+ are the creation operators of the transitions $|g\rangle_1 \leftrightarrow |e\rangle_1$ and $|g\rangle_2 \leftrightarrow |e\rangle_2$ of the qubits q_1 and q_2 , respectively. $|g\rangle_{1(2)}$ and $|e\rangle_{1(2)}$ are the ground and the first excited states of the qubit $q_{1(2)}$, respectively. g_1^a and g_2^b are the coupling strength between q_1 and r_a and that between q_2 and r_b , respectively. $g_{f,j}^l$ is the coupling strength between $r_{a(b)}$ and the mode j of the TL r_f . ϕ is the phase induced by the propagating field through the TL r_f of length l with the relation $\phi = 2\pi\omega l/c$ in which c is the speed of light.

In the short TL limit $(2L\kappa_f^{a(b)})/(2\pi c) \leq 1$, only one resonant mode f of the TL r_f interacts with the resonators' modes (L is the length of r_f and $\kappa_f^{a(b)}$ is the decay rate of the resonator $r_{a(b)}$ into a continuum of TL modes) [54]. The Hamiltonian H can be reduced to

$$\begin{aligned} H_{int} &= g_1^a(a^\dagger \sigma_1^- e^{-i\delta_1^a t} + a \sigma_1^+ e^{i\delta_1^a t}) + g_2^b(b^\dagger \sigma_2^- e^{-i\delta_2^b t} + b \sigma_2^+ e^{i\delta_2^b t}) \\ &\quad + g_f^a(f^\dagger a + f a^\dagger) + g_f^b(f^\dagger b + f b^\dagger). \end{aligned} \quad (2)$$

In the Schrödinger picture, this Hamiltonian can be rewritten as

$$\begin{aligned} H' &= \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_f f^\dagger f + \omega_1 \sigma_1^+ \sigma_1^- + \omega_2 \sigma_2^+ \sigma_2^- \\ &\quad + g_1^a(a^\dagger \sigma_1^- + a \sigma_1^+) + g_2^b(b^\dagger \sigma_2^- + b \sigma_2^+) \\ &\quad + g_f^a(f^\dagger a + f a^\dagger) + g_f^b(f^\dagger b + f b^\dagger). \end{aligned} \quad (3)$$

To generate and construct the Bell state and the c-phase gate on the remote transmon qutrits below, we consider the all-resonance condition with $\omega_a = \omega_b = \omega_f = \omega_1 = \omega_2 = \omega$ by letting the frequencies of the qubits and the resonators and the TL equal to each other and $g_f^a = g_f^b = g$ by letting the coupling strength between r_a and r_f equal to the one of r_b and r_f . If one takes the canonical transformations $C_\pm = \frac{1}{2}(a + b \pm \sqrt{2}f)$ and $C = \frac{\sqrt{2}}{2}(a - b)$ [58, 59], the Hamiltonian H' can be represented as

$$\begin{aligned} H'' &= \omega \sigma_1^+ \sigma_1^- + \omega \sigma_2^+ \sigma_2^- + \omega C^\dagger C + (\omega + \sqrt{2}g) C_+ C_+^\dagger \\ &\quad + (\omega - \sqrt{2}g) C_- C_-^\dagger + \frac{1}{2} [g_1^a (C_+ + C_- + \sqrt{2}C) \sigma_1^+ \\ &\quad + g_1^a (C_+^\dagger + C_-^\dagger + \sqrt{2}C^\dagger) \sigma_1^- + g_2^b (C_+ + C_- - \sqrt{2}C) \sigma_2^+ \\ &\quad + g_2^b (C_+^\dagger + C_-^\dagger - \sqrt{2}C^\dagger) \sigma_2^-]. \end{aligned} \quad (4)$$

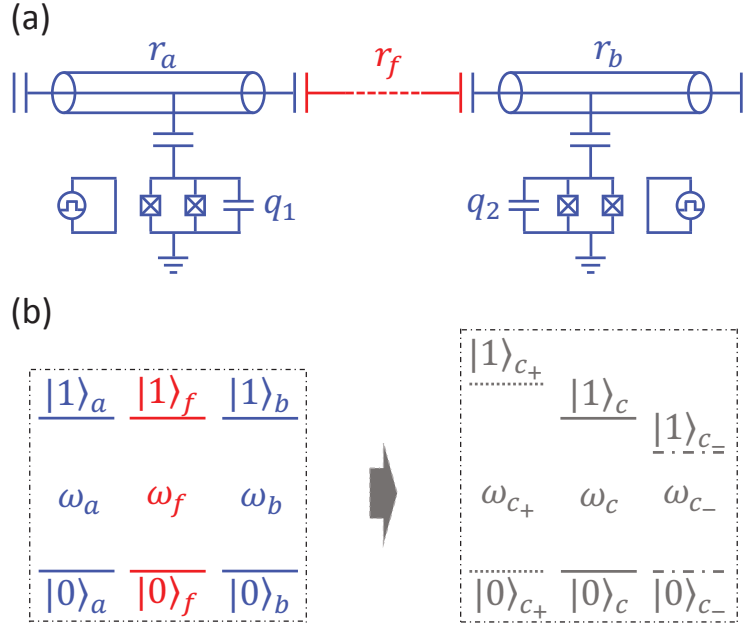


FIG. 1: (a) Setup for a distributed quantum computing composed of two remote qubits q_1 and q_2 trapped in different resonators r_a and r_b connected by a transmission line r_f . (b) Illustrations of the energy splitting of the subsystem composed of r_a , r_b , and r_f .

Here the modes C and C_{\pm} are three bosonic modes and they are not coupled to each other. From Eq. (4), the energy level of the subsystem composed of r_a , r_b , and r_f are split into three different parts with frequencies ω_{c+} , ω_{c-} , and ω_c signed by the modes C_+ , C_- , and C , respectively, as shown in Fig. 1 (b). Because of the contributions of the fields of r_a and r_b , the three modes C and C_{\pm} interact with the two qubits q_1 and q_2 . When $g \gg \{g_1^a, g_2^b\}$, the excitations of modes C_{\pm} are highly suppressed as $\omega \pm \sqrt{2}g$ detune with the resonance modes (C , q_1 , and q_2 with frequency of ω) largely, which means the modes C_{\pm} are the dark ones to the frequency ω_f of r_f , and the Hamiltonian H'' can be reduced to

$$H''' = \omega\sigma_1^+\sigma_1^- + \omega\sigma_2^+\sigma_2^- + \omega C^+C + \frac{1}{\sqrt{2}} \left[g_1^a (C\sigma_1^+ + C^+\sigma_1^-) - g_2^b (C\sigma_2^+ + C^+\sigma_2^-) \right]. \quad (5)$$

It can be written as

$$H_{eff} = \frac{1}{\sqrt{2}} \left[g_1^a (C\sigma_1^+ + C^+\sigma_1^-) - g_2^b (C\sigma_2^+ + C^+\sigma_2^-) \right] \quad (6)$$

in the interaction picture. Here, only the mode $C = \frac{\sqrt{2}}{2}(a - b)$ is left, which means that the TL can not be populated in the all-resonance regime in our system. The interaction between two remote two-energy-level qubits expressed by H_{eff} will be used to generate and construct the all-resonance Bell state and c-phase gate on the two remote qutrits below.

III. C-PHASE GATE ON THE TWO REMOTE QUTRITS q_1 AND q_2

With the two transitions only discussed in the section II, one cannot construct the one-step all-resonance c-phase gate on the two remote transmon qutrits because of the existing of the states $|e\rangle_1$ and $|e\rangle_2$ according to Eq. (6).

Here, we consider the second excited energy level $|s\rangle_2$ of q_2 and take $\omega_{1:ge}/(2\pi) = \omega_{2:es}/(2\pi) = \omega_a/(2\pi) = \omega_b/(2\pi) = \omega_f/(2\pi)$. The illustrations of the interactions between q_1 and r_a , r_f and r_a (r_b), and q_2 and r_b for constructing the c-phase gate on q_1 and

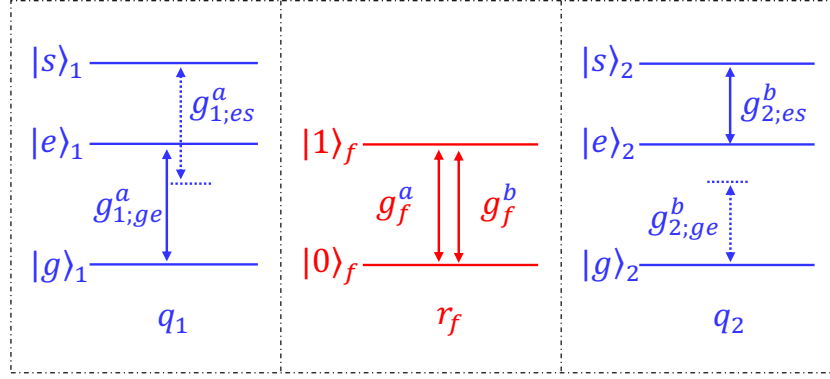


FIG. 2: Illustrations of interactions between q_1 and r_a , r_f and r_a (r_b), and q_2 and r_b , respectively, for the construction of the c-phase gate on two remote transmon qutrits q_1 and q_2 .

q_2 are shown in Fig. 2. The Hamiltonian of the whole system can be expressed as

$$\begin{aligned}
 H_{2q} = & g_{1;ge}^a (a^\dagger \sigma_{1;ge}^- e^{-i\delta_{1;ge}^a t} + a \sigma_{1;ge}^+ e^{i\delta_{1;ge}^a t}) \\
 & + g_{1;es}^a (a^\dagger \sigma_{1;es}^- e^{-i\delta_{1;es}^a t} + a \sigma_{1;es}^+ e^{i\delta_{1;es}^a t}) \\
 & + g_{2;ge}^b (b^\dagger \sigma_{2;ge}^- e^{-i\delta_{2;ge}^b t} + b \sigma_{2;ge}^+ e^{i\delta_{2;ge}^b t}) \\
 & + g_{2;es}^b (b^\dagger \sigma_{2;es}^- e^{-i\delta_{2;es}^b t} + b \sigma_{2;es}^+ e^{i\delta_{2;es}^b t}) \\
 & + g_f^a (f^\dagger a + f a^\dagger) + g_f^b (f^\dagger b + f b^\dagger),
 \end{aligned} \tag{7}$$

in which $\sigma_{1(2);ge}^+$ and $\sigma_{1(2);es}^+$ are the creation operators of the transitions $|g\rangle_{1(2)} \leftrightarrow |e\rangle_{1(2)}$ and $|e\rangle_{1(2)} \leftrightarrow |s\rangle_{1(2)}$ of the qutrit $q_{1(2)}$, respectively. $g_{1(2);ge}^{a(b)}$ and $g_{1(2);es}^{a(b)}$ ($g_{1(2);es}^{a(b)} = \sqrt{2}g_{1(2);ge}^{a(b)}$) are the coupling strengths between the two transitions of $q_{1(2)}$ and $r_{a(b)}$, respectively. $\delta_{1(2);ge}^{a(b)} = \omega_{1(2);ge} - \omega_{a(b)}$ and $\delta_{1(2);es}^{a(b)} = \omega_{1(2);es} - \omega_{a(b)}$. $\omega_{1(2);ge}$ ($\omega_{1(2);ef}$) is the frequency of the transition $|g\rangle_{1(2)} \leftrightarrow |e\rangle_{1(2)}$ ($|e\rangle_{1(2)} \leftrightarrow |s\rangle_{1(2)}$) of the qutrit $q_{1(2)}$. $|s\rangle_{1(2)}$ is the second excited states of $q_{1(2)}$. By taking $\omega_{1(2);ge} - \omega_{1(2);es} \gg \{g_{1;ge}^a, g_{2;ge}^b\}$, the Hamiltonian H_{2q} can be reduced to

$$\begin{aligned}
 H'_{2q} = & g_{1;ge}^a (a^\dagger \sigma_{1;ge}^- + a \sigma_{1;ge}^+) + g_{2;es}^b (b^\dagger \sigma_{2;es}^- + b \sigma_{2;es}^+) \\
 & + g_f^a (f^\dagger a + f a^\dagger) + g_f^b (f^\dagger b + f b^\dagger),
 \end{aligned} \tag{8}$$

in which the dispersive coupling between the transition $|e\rangle_1 \leftrightarrow |s\rangle_1$ of the qutrit q_1 and r_a and the one between the transition $|g\rangle_2 \leftrightarrow |e\rangle_2$ of the qutrit q_2 and r_b are ignored.

Taking the same canonical transformations as the ones in Sec. II and $g_f^a = g_f^b \gg \{g_{1;ge}^a, g_{2;es}^b\}$, the Hamiltonian H'_{2q} becomes

$$H'_{eff} = \frac{1}{\sqrt{2}} \left[g_{1;ge}^a (C \sigma_{1;ge}^+ + C^\dagger \sigma_{1;ge}^-) - g_{2;es}^b (C \sigma_{2;es}^+ + C^\dagger \sigma_{2;es}^-) \right]. \tag{9}$$

Suppose that $|\psi_1\rangle = |g\rangle_1 |g\rangle_2 |0\rangle_c$, $|\psi_2\rangle = |g\rangle_1 |e\rangle_2 |0\rangle_c$, $|\psi_3\rangle = |e\rangle_1 |g\rangle_2 |0\rangle_c$, and $|\psi_4\rangle = |e\rangle_1 |e\rangle_2 |0\rangle_c$ ($|0\rangle_c \equiv |0\rangle_a |0\rangle_b |0\rangle_f$) are the initial states of the system undergoes the Hamiltonian H'_{eff} , respectively, one can get their evolutions as

$$|\Psi_1(t)\rangle = e^{iH'_{eff}t} |g\rangle_1 |g\rangle_2 |0\rangle_c = |g\rangle_1 |g\rangle_2 |0\rangle_c, \tag{10}$$

$$|\Psi_2(t)\rangle = e^{-iH'_{eff}t} |g\rangle_1 |e\rangle_2 |0\rangle_c = |g\rangle_1 |e\rangle_2 |0\rangle_c, \tag{11}$$

$$\begin{aligned}
 |\Psi_3(t)\rangle &= e^{-iH'_{eff}t} |e\rangle_1 |g\rangle_2 |0\rangle_c \\
 &= \cos\left(\frac{g_{1;ge}^a}{\sqrt{2}}t\right) |e\rangle_1 |g\rangle_2 |0\rangle_c + \sin\left(\frac{g_{1;ge}^a}{\sqrt{2}}t\right) |g\rangle_1 |g\rangle_2 |1\rangle_c,
 \end{aligned} \tag{12}$$

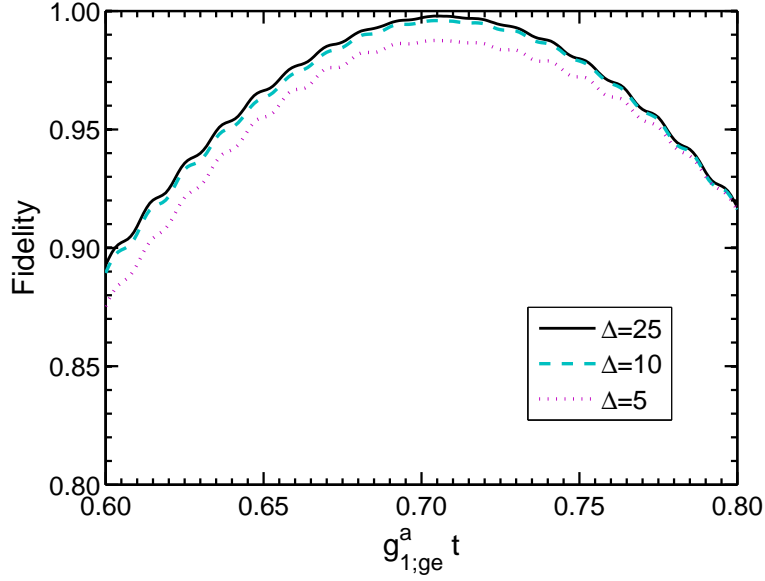


FIG. 3: The fidelity of our c-phase gate varies with gt and different $\Delta = g_f^{a(b)}/g_{1;ge}^a$.

$$\begin{aligned}
 |\Psi_4(t)\rangle &= e^{iH'_{eff}t} |e\rangle_1 |e\rangle_2 |0\rangle_c \\
 &= \frac{1}{G'} \left[(g_{2;es}^b)^2 + (g_{1;ge}^a)^2 \cos\left(\sqrt{\frac{G'}{2}}t\right) \right] |e\rangle_1 |e\rangle_2 |0\rangle_c \\
 &\quad - \frac{g_{1;ge}^a g_{2;es}^b}{G'} \left[\cos\left(\sqrt{\frac{G'}{2}}t\right) - 1 \right] |g\rangle_1 |s\rangle_2 |0\rangle_c \\
 &\quad - \frac{ig_{1;ge}^a}{\sqrt{G'}} \sin\left(\sqrt{\frac{G'}{2}}t\right) |g\rangle_1 |e\rangle_2 |1\rangle_c,
 \end{aligned} \tag{13}$$

where $G' = (g_{1;ge}^a)^2 + (g_{2;es}^b)^2$. From the evolutions of the four states, one can construct the c-phase gate on the two remote qutrits q_1 and q_2 . In detail, we suppose the initial state of the system described by H'_{eff} is

$$|\Psi_0^{cp}\rangle = (\cos\theta_1 |g\rangle_1 + \sin\theta_1 |e\rangle_1) \otimes (\cos\theta_2 |g\rangle_2 + \sin\theta_2 |e\rangle_2) \otimes |0\rangle_c. \tag{14}$$

According to Eqs. (10) and (11), one can keep the states $|g\rangle_1 |g\rangle_2 |0\rangle_c$ and $|g\rangle_1 |e\rangle_2 |0\rangle_c$ unchanged. By taking the proper $g_{1;ge}^a$ and $g_{2;es}^b$ to satisfy $\frac{g_{1;ge}^a}{\sqrt{2}}t = (2k-1)\pi$ and $\sqrt{\frac{G'}{2}}t = 2m\pi$ ($k, m = 1, 2, 3, \dots$) simultaneously, one can achieve the condition that when the state $|e\rangle_1 |g\rangle_2 |0\rangle_c$ undergoes an odd number of periods and generates a minus phase (from Eq. (12)), the state $|e\rangle_1 |e\rangle_2 |0\rangle_c$ undergoes an even number of periods and keeps unchanged (from Eq. (13)) meanwhile. That is, the state of the system evolves from $|\Psi_0^{cp}\rangle$ to the final state

$$|\Psi_f^{cp}\rangle = (\alpha_1 |g\rangle_1 |g\rangle_2 + \alpha_2 |g\rangle_1 |e\rangle_2 - \alpha_3 |e\rangle_1 |g\rangle_2 + \alpha_4 |e\rangle_1 |e\rangle_2) \otimes |0\rangle_c, \tag{15}$$

which is just the target state after our c-phase gate operation on q_1 and q_2 with the initial state $|\Psi_0^{cp}\rangle$. Here $\alpha_1 = \cos\theta_1 \cos\theta_2$, $\alpha_2 = \cos\theta_1 \sin\theta_2$, $\alpha_3 = \sin\theta_1 \cos\theta_2$, and $\alpha_4 = \sin\theta_1 \sin\theta_2$. In the basis $\{|g\rangle_1 |g\rangle_2, |g\rangle_1 |e\rangle_2, |e\rangle_1 |g\rangle_2, |e\rangle_1 |e\rangle_2\}$, the matrix of the c-phase gate is

$$U^{cp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

To get the c-phase gate within a short time, we take $k = m = 1$, that is, $g_{2;es}/(2\pi) = \sqrt{2}g_{2;ge}/(2\pi) = \sqrt{3}g_{1;ge}/(2\pi)$. Supposing that the initial state of the system is $|\Psi_{max}\rangle = \frac{1}{2}(|\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle + |\psi_4\rangle)$, one can get the state $|\Psi_{max}^{cp}\rangle = \frac{1}{2}(|\psi_1\rangle + |\psi_2\rangle - |\psi_3\rangle + |\psi_4\rangle)$

after our c-phase gate operation on the two remote qutrits q_1 and q_2 with a maximal fidelity of 99.8% ($\Delta = 25$), 99.6% ($\Delta = 10$), and 98.8% ($\Delta = 5$) within $gt = 0.705$ ($\omega_{1;ge} - \omega_{1;es} = \omega_{2;ge} - \omega_{2;es} = 90g_{1;ge}^a$), by using the definition

$$F_{max}^{cp} = |\langle \Psi_{max}^{cp} | e^{-iH_{2q}t} | \Psi_{max} \rangle|^2, \quad (17)$$

as shown in Fig. 3. Here $\Delta \equiv g_f^{a(b)} / g_{1;ge}^a$.

IV. POSSIBLE EXPERIMENTAL IMPLEMENTATION AND FIDELITY

In experiment, a high quality factor $Q \sim 2 \times 10^6$ of a 1D SR has been demonstrated [60]. By considering the relation between κ , Q , and the frequency of resonator ω_r with $\kappa = \omega_r / Q$ [17], the best life time of a microwave photon in a superconducting resonator can reach $\sim 50 \mu s$. The coherence time of a transmon qubit [61–64] can also reach $50 \mu s$ by using titanium nitride [65]. The tunable range of the transition frequency of a transmon qubit can reach 2.5 GHz, which helps us to tune our transmon qutrits to resonate (detune) with their corresponding resonator (largely) effectively. The coupling strength between a transmon qutrit and a SR can be realized larger than 200 MHz [66]. The anharmonicity between the two transitions of a transmon qutrit can reach $\omega_{1;ge} / (2\pi) - \omega_{1;es} / (2\pi) = \omega_{2;ge} / (2\pi) - \omega_{2;es} / (2\pi) = 0.72$ GHz [67], which lets us ignore the detune interaction between each qutrit and their corresponding resonator, compared with the small coupling strength between them. As for the SRs and the TL r_f , one can couple them by using the SQUID, which can reach a coupling strength of $g_f^{a(b)} / (2\pi) \sim 200$ MHz theoretically with reasonable experimental parameters [68]. Moreover, one can also use the capacitance coupling between resonators and the superconducting TL. With the reasonable parameters $\omega_a / (2\pi) = \omega_b / (2\pi) = \omega_f / (2\pi) = 6$ GHz, the coupling capacitance $C = 13.3$ fF, and the capacitance per unit length of the transmission line and the resonators $C_r = 2$ pF [17, 69], the capacitance coupling strength can reach $g_f^{a(b)} / (2\pi) = 40$ MHz (which will be discussed below for the construction of the c-phase gate with $\Delta = 5$). It is worth noticing that a coupling strength between a SR and a superconducting TL has been realized with about 32 MHz [70].

To show the feasibility of our scheme for the construction of the c-phase gate on two remote qutrits, we numerically simulate the fidelity of the scheme based on the parameters realized in experiments or predicted theoretically with reasonable experimental parameters.

The dynamics of the quantum system undergoes the Hamiltonian H_{2q} is determined by the master equation

$$\begin{aligned} \frac{d\rho}{dt} = & -i[H_{2q}, \rho] + \kappa_a D[a]\rho + \kappa_b D[b]\rho + \kappa_f D[f]\rho \\ & + \sum_{l=1,2} \{ \gamma_{l;ge} D[\sigma_{l;ge}^-] \rho + \gamma_{l;es} D[\sigma_{l;es}^-] \rho \\ & + \gamma_{l;e}^\phi (\sigma_{l;ee} \rho \sigma_{l;ee} - \sigma_{l;ee} \rho / 2 - \rho \sigma_{l;ee} / 2) \\ & + \gamma_{l;s}^\phi (\sigma_{l;ss} \rho \sigma_{l;ss} - \sigma_{l;ss} \rho / 2 - \rho \sigma_{l;ss} / 2) \}. \end{aligned} \quad (18)$$

Here, $\kappa_{a,b,f}$ is the decay rate of the resonator $r_{a,b,f}$. $\gamma_{l;ge}$ ($\gamma_{l;es}$) and $\gamma_{l;e}^\phi$ ($\gamma_{l;s}^\phi$) are the energy relaxation and the dephase rates of the transition $|e\rangle_l \leftrightarrow |g\rangle_l$ ($|s\rangle_l \leftrightarrow |e\rangle_l$) of the transmon qutrits q_l ($l = 1, 2$), respectively. $\gamma_{l;ge}^{-1} = (\gamma_{l;ge}^\phi)^{-1} = (\gamma_{l;es}^\phi)^{-1} = 2\gamma_{l;es}^{-1}$ [71], $\sigma_{l;ee} = |e\rangle_l \langle e|$, and $\sigma_{l;ss} = |s\rangle_l \langle s|$. $D[L]\rho = (2L\rho L^\dagger - L^\dagger L\rho - \rho L^\dagger L) / 2$. Because of the competition relation between the coupling strength $g_{1;ge}^a$ and the decay rates and that between the decoherence time of resonators and qutrits, for different $\gamma_{l;ge}$ and $\kappa_{a,b,f}$, one should choose different $g_{1;ge}^a$ to reach the maximal fidelity of our scheme for constructing the c-phase gate on remote qutrits q_1 and q_2 . Besides, Δ for the simulation of our scheme below are both larger than 15, which indicates there are almost no MP in the TL and the influence from the decay rate κ_f of r_f is not considered here. The coupling strengths $g_{1;ge}^a / (2\pi)$ chosen below for the c-phase gate construction are the optimal ones which correspond to the highest fidelities of the gate when we fix $g_f^{a(b)} / (2\pi) = 200$ MHz and $\gamma_{l;ge}^{-1} = \kappa_{a,b,f} = 50 \mu s$ by considering the set of discretized $g_{1;ge}^a$ values, varying from 1 to 100 MHz in steps of 1 MHz.

To show the feasibility of our c-phase gate on remote qutrits q_1 and q_2 with decoherence time and the decay time of the qutrits and the resonators, we numerically simulate the fidelity of $|\Psi_{max}^{cp}\rangle$ after our c-phase gate operations on the whole system (the initial state of the system is $|\Psi_{max}\rangle$) by using the definition

$$F_{cp} = \langle \Psi_{max}^{cp} | \rho(t) | \Psi_{max}^{cp} \rangle, \quad (19)$$

in which the effects from the unresonant parts

$$H_1 = g_{a,1}^{e,f} (a\sigma_{1;e,f}^+ e^{i\delta_{a,1}^{e,f}t} + a^\dagger \sigma_{1;e,f}^- e^{-i\delta_{a,1}^{e,f}t}) \quad (20)$$

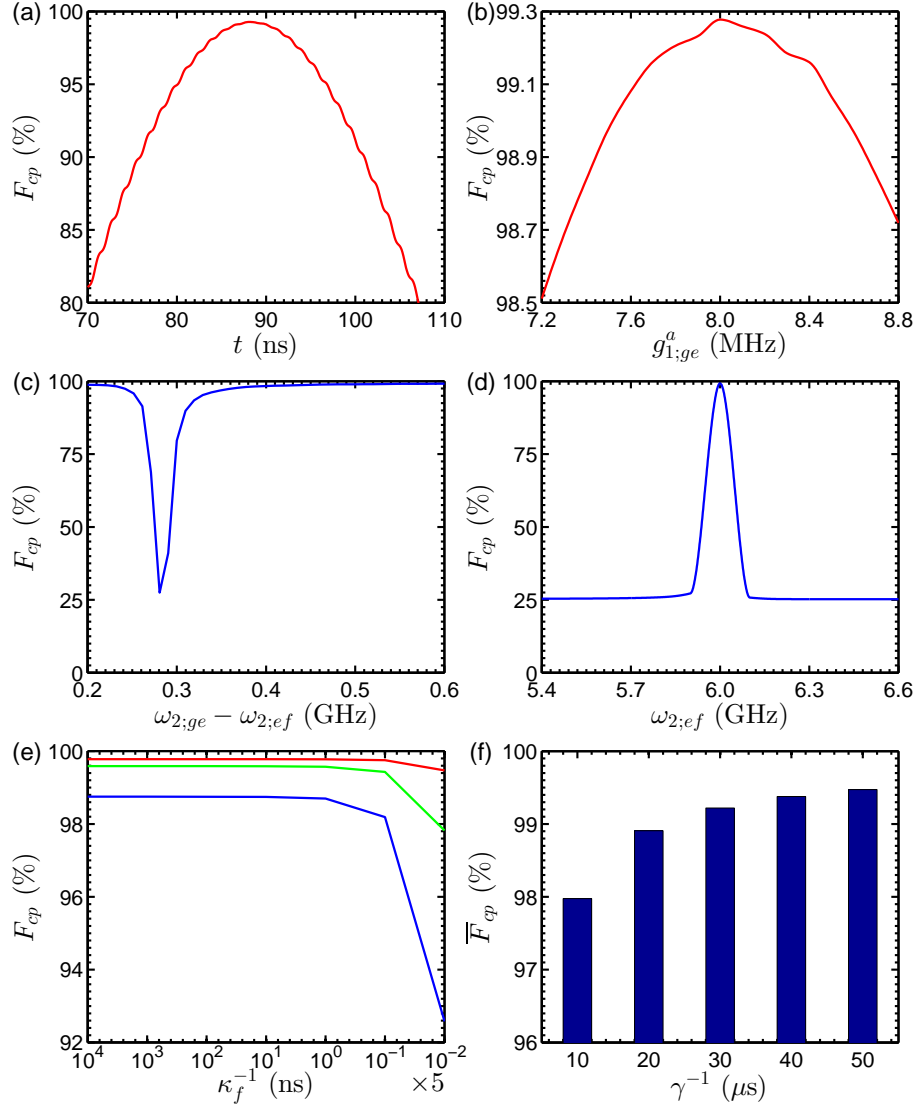


FIG. 4: (a) The fidelity of the c-phase gate on q_1 and q_2 varies with the operation time t . (b)-(f) The relations between the fidelity of the gate and $g_{1;ge}^a$, $\omega_{2;ge} - \omega_{2;ef}$, $\omega_{2;ef}$, κ_f^{-1} , and γ^{-1} , respectively.

and

$$H_2 = g_{b,2}^{g,e} (b\sigma_{2;g,e}^+ e^{i\delta_{b,2}^{g,e}t} + b^+\sigma_{2;g,e}^- e^{-i\delta_{b,2}^{g,e}t}) \quad (21)$$

are considered. Parameters chosen here are $\omega_{1;ge}/(2\pi) = \omega_{2;es}/(2\pi) = \omega_a/(2\pi) = \omega_b/(2\pi) = \omega_f/(2\pi) = 6$ GHz, $\sqrt{\frac{2}{3}}g_{2;ge}^b/(2\pi) = g_{1;ge}^a/(2\pi) = 8$ MHz. $\gamma_{1;ge}^{-1} = \kappa_{a,b,f}^{-1} = 50$ μ s. As shown in Fig. 4 (a), the fidelity of the state $|\Psi_{max}^{cp}\rangle$ can reach 99.28% within 88.1 ns.

In the realistic experiment, parameters of the system can not match the ones accurately chosen above. We give the influences on the fidelity of the state $|\Psi_{max}^{cp}\rangle$ from the coupling strength, the anharmonicity, the decoherence time, and the frequency of the qutrits and the decay time of resonators as shown in Fig. 4 (b)-(f). In each figure in Fig. 4, parameters are kept unchanged except for the one signed in the axis of abscissas. The influences from $g_{1;ge}^a$ is shown in Fig. 4 (b) and the small change of $g_{1;ge}^a$ influences the fidelity little. Fig. 4 (c) indicates that the anharmonicity of q_2 should be chosen to let the transition frequency $\omega_{2;ge}$ detune with $\omega_f + \sqrt{2}g_f^{a(b)}$ largely, which is required when we reduce the Hamiltonian from H_{2q} to H_{eff}'' . Accurate resonance between the two remote qutrits is required as shown in Fig. 4 (d). In the large-scale integration of our system, the interaction between qutrits can be turned off conveniently by tuning the frequency of the qutrits. In Fig. 4 (e), we give the influences on the fidelity of the state from the decay time of r_f . It can be seen that when $\kappa_f^{-1} > 10$ ns, κ_f^{-1} influences the fidelity of the state little.

To show the possible influence from the decay rates of the resonators and the decoherence time of the qutrits, we calculate the average gate fidelity of the c-phase gate with different $\Gamma^{-1} = \gamma_{l;ge}^{-1} = \kappa_{a,b,f}^{-1}$, shown in Fig. 4 (f) by using the average-gate-fidelity definition

$$F = \left(\frac{1}{2\pi}\right)^2 \int_0^{2\pi} \int_0^{2\pi} \langle \Psi_f^{cp} | \rho(t) | \Psi_f^{cp} \rangle d\theta_1 d\theta_2. \quad (22)$$

Here, $\rho(t)$ is the realistic density operator after our c-phase gate operation on the initial state Ψ_0^{cp} with the Hamiltonian H . It is worth noticing that the decay time $20 \mu\text{s}$ corresponds to the typically quality factor $Q \sim 7 \times 10^5$ of a 1D superconducting resonator [72] and the coherence time $20 \mu\text{s}$ of a transmon qutrit can also be readily realized in experiment [64]. Although the coupling strength $g_f^{a(b)}/(2\pi)$ taken here is 200 MHz (predicted theoretically in [68]) which satisfies $\Delta = 25$ and it has not been realized in experiments, if we take $\Delta = 5$ ($g_f^{a(b)}/(2\pi) = 40$ MHz) and the more of the actual situation of the life time of SRs and qutrits with $\Gamma^{-1} = 20 \mu\text{s}$, the fidelity of our c-phase gate can also reach a high value of 98% (compared with the fidelities between $\Delta = 25$ and $\Delta = 5$ as shown in Fig. 3) which should be enhanced further by taking corresponding optimal parameters. Corresponding to the operation time of the c-phase gate construction, the length of the superconducting TL can, in principle, reach the scale of several meters.

V. DISCUSSION AND SUMMARY

On one hand, in order to use the dark photons in the TL to achieve the c-phase gate on qutrits q_1 and q_2 , one should take $g_{1(2);ge}^{a(b)} \ll g_f^{a(b)}$. On the other hand, the small coupling strength of $g_{1(2);ge}^{a(b)}$ does not require the large anharmonicities of the qutrits. Moreover, to achieve our c-phase scheme, the Ξ -type energy level of the qutrits is required. Besides the transmon qutrit, the superconducting charge qutrit [73, 74] or phase qutrit [75] can also be applied to our scheme. By using the transmon qutrit or the phase qutrit with $\omega_{ge}/(2\pi) > \omega_{ef}/(2\pi)$, one should take the proper anharmonicity of q_2 to let the transition frequency $\omega_{2;ge}$ detune with $\omega_f + \sqrt{2}g_f^{a(b)}$ largely. By using the charge qutrit with $\omega_{ge}/(2\pi) < \omega_{ef}/(2\pi)$, one should take $\omega_{2;ge}$ detune with $\omega_f - \sqrt{2}g_f^{a(b)}$ largely. That is, when the frequency $\omega_{2;ge} \sim \omega_f + \sqrt{2}g_f^{a(b)}$, the effective Hamiltonians H_{eff} and H'_{eff} can not be obtained as the mode C_{\pm} can not be suppressed effectively.

In summary, we have proposed a one-step scheme to achieve the c-phase gate on two remote transmon qutrits trapped in different resonators connected by a superconducting TL. The scheme works in the all-resonance regime with just one step, which leads to a fast operation and can be demonstrated in experiment easily. Moreover, our scheme is robust against the TL loss by using the dark microwave photon. That is, the superconducting TL needs not to be populated, which is convenient to be extended to a large-scale integration condition by the complex designation of a long-length TL to link lots of remote circuit QEDs.

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- [1] Nilsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
 - [2] Shor P W 1997 *SIAM J. Sci. Statist. Comput.* **26** 1484
 - [3] Grover L K 1997 *Phys. Rev. Lett.* **79** 325
 - [4] Long G L 2001 *Phys. Rev. A* **64** 022307
 - [5] Knill E, Laflamme R and Milburn G J 2001 *Nature* **409** 46
 - [6] O'Brien J L, Pryde G J, White A G, Ralph T C and Branning D 2003 *Nature* **426** 264
 - [7] Ren B C and Deng F G 2014 *Sci. Rep.* **4** 4623
 - [8] Ren B C, Wang G Y and Deng F G 2015 *Phys. Rev. A* **91** 032328
 - [9] Wei H R and Deng F G 2013 *Opt. Express* **21** 17671
 - [10] Jones J A, Mosca M and Hansen R H 1998 *Nature* **393** 344
 - [11] Long G L and Xiao L 2003 *J. Chem. Phys.* **119** 8473
 - [12] Feng G R, Xu G F and Long G L 2013 *Phys. Rev. Lett.* **110** 190501

- [13] Togan E, Chu Y, Trifonov A S, Jiang L, Maze J, Childress L, Dutt M V G, Sørensen A S, Hemmer P R, Zibrov A S and Lukin M D 2010 *Nature* **466** 730
- [14] Yang W L, Yin Z Q, Xu Z Y, Feng M and Du J F 2010 *Appl. Phys. Lett.* **96** 241113
- [15] Wei H R and Deng F G 2013 *Phys. Rev. A* **88** 042323
- [16] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University press)
- [17] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev. A* **69** 062320
- [18] Wallraff A, Schuster D I, Blais A, Frunzio L, Huang R S, Majer J, Kumar S, Girvin S M and Schoelkopf R J 2004 *Nature* **431** 162
- [19] Barends R, Kelly J, Megrant A, Veitia A, Sank D, Jeffrey E, White T C, Mutus J, Fowler A G, Campbell B, Chen Y, Chen Z, Chiaro B, Dunsworth A, Neill C, O'Malley P, Roushan P, Vainsencher A, Wenner J, Korotkov A N, Cleland A N and Martinis J M 2014 *Nature* **508** 500
- [20] DiCarlo L, Chow J M, Gambetta J M, Bishop L S, Johnson B R, Schuster D I, Majer J, Blais A, Frunzio L, Girvin S M and Schoelkopf R J 2009 *Nature* **460** 240
- [21] Haack G, Helmer F, Mariani M, Marquardt F and Solano E 2010 *Phys. Rev. B* **82** 024514
- [22] Strauch F W 2011 *Phys. Rev. A* **84** 052313
- [23] Hua M, Tao M J and Deng F G 2014 *Phys. Rev. A* **90** 012328
- [24] Hua M, Tao M J and Deng F G 2015 *Sci. Rep.* **5** 9274
- [25] McKay D C, Naik R, Reinhold P, Bishop L S and Schuster D I 2015 *Phys. Rev. Lett.* **114** 080501
- [26] Steffen M, Ansmann M, Bialczak R C, Katz N, Lucero E, McDermott R, Neeley M, Weig E M, Cleland A N and Martinis J M 2006 *Science* **313** 1423
- [27] Cao Y, Huo W Y, Ai Q and Long G L 2011 *Phys. Rev. A* **84** 053846
- [28] Leghtas Z, Vool U, Shankar S, Hatridge M, Girvin S M, Devoret M H and Mirrahimi M 2013 *Phys. Rev. A* **88** 023849
- [29] Strauch F W 2012 *Phys. Rev. Lett.* **109** 210501
- [30] Strauch F W, Onyango D, Jacobs K and Simmonds R W 2012 *Phys. Rev. A* **85** 022335
- [31] Wallraff A, Schuster D I, Blais A, Frunzio L, Majer J, Devoret M H, Girvin S M and Schoelkopf R J 2005 *Phys. Rev. Lett.* **95** 060501
- [32] Johnson B R, Reed M D, Houck A A, Schuster D I, Bishop L S, Ginossar E, Gambetta J M, DiCarlo L, Frunzio L, Girvin S M and Schoelkopf R J 2010 *Nat. Phys.* **6** 663
- [33] Feng W, Wang P Y, Ding X M, Xu L T and Li X Q 2011 *Phys. Rev. A* **83** 042313
- [34] Majer J, Chow J M, Gambetta J M, Koch J, Johnson B R, Schreier J A, Frunzio L, Schuster D I, Houck A A, Wallraff A, Blais A, Devoret M H, Girvin S M and Schoelkopf R J 2007 *Nature* **449** 443
- [35] Hua M, Tao M J and Deng F G 2015 arXiv:1507.00069.
- [36] Wu C W, Gao M, Li H Y, Deng Z J, Dai H Y, Chen P X and Li C Z 2012 *Phys. Rev. A* **85** 042301
- [37] Yang C P, Su Q P and Han S Y 2012 *Phys. Rev. A* **86** 022329
- [38] Yang C P, Su Q P, Zheng S B and Nori F 2016 *Phys. Rev. A* **93** 042307
- [39] Pellizzari T 1997 *Phys. Rev. Lett.* **79** 5242
- [40] van Loo A F, Fedorov A, Lalumière K, Sanders B C, Blais A and Wallraff A 2013 *Science* **342** 1494
- [41] Yin Y, Chen Y, Sank D, O'Malley P J J, White T C, Barends R, Kelly J, Lucero E, Mariani M, Megrant A, Neill C, Vainsencher A, Wenner J, Korotkov A N, Cleland A N and Martinis J M 2013 *Phys. Rev. Lett.* **110** 107001
- [42] Srinivasan S J, Sundaresan N M, Sadri D, Liu Y, Gambetta J M, Yu T, Girvin S M and Houck A A 2014 *Phys. Rev. A* **89** 033857
- [43] Pechal M, Huthmacher L, Eichler C, Zeytinoglu S, Abdumalikov A A, Berger J S, Wallraff A and Filipp S 2014 *Phys. Rev. X* **4** 041010
- [44] Roch N, Schwartz M E, Motzoi F, Macklin C, Vijay R, Eddins A W, Korotkov A N, Whaley K B, Sarovar M and Siddiqi I 2014 *Phys. Rev. Lett.* **112** 170501
- [45] Mandt S, Sadri D, Houck A A and Türeci H E 2015 *New J. Phys.* **17** 053018
- [46] Cirac J I, Ekert A K, Huelga S F and Macchiavello C 1999 *Phys. Rev. A* **59** 4249
- [47] Clark S, Peng A, Gu M and Parkins S 2003 *Phys. Rev. Lett.* **91** 177901
- [48] Browne D E, Plenio M B and Huelga S F 2003 *Phys. Rev. Lett.* **91** 067901
- [49] Duan L M and Kimble H J 2003 *Phys. Rev. Lett.* **90**, 253601
- [50] Mancini S and Bose S 2005 *Phys. Rev. A* **70** 022307
- [51] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221
- [52] Xiao Y F, Lin X M, Gao J, Yang Y, Han Z F and Guo G C 2004 *Phys. Rev. A* **70** 042314
- [53] Lü X Y, Wu J, Zheng L L and Zhan Z M 2011 *Phys. Rev. A* **83** 042302
- [54] Yin Z Q and Li F L 2007 *Phys. Rev. A* **75** 012324
- [55] Clader B D 2014 *Phys. Rev. A* **90** 012324
- [56] Yin Z Q, Yang W L, Sun L Y and Duan L M 2015 *Phys. Rev. A* **91** 012333
- [57] van Enk S J, Kimble H J, Cirac J I and Zoller P 1999 *Phys. Rev. A* **59** 2659
- [58] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503
- [59] Yang W L, Hu Y, Yin Z Q, Deng Z J and Feng M 2011 *Phys. Rev. A* **83** 022302
- [60] Megrant A, Neill C, Barends R, Chiaro B, Chen Y, Feigl L, Kelly J, Lucero E, Mariani M, O'Malley P J J, Sank D, Vainsencher A, Wenner J, White T C, Yin Y, Zhao J, Palmstrøm C J, Martinis J M and Cleland A N 2012 *Appl. Phys. Lett.* **100** 113510
- [61] Koch J, Yu T M, Gambetta J, Houck A A, Schuster D I, Majer J, Blais A, Devoret M H, Girvin S M and Schoelkopf R J 2007 *Phys. Rev. A* **76** 042319
- [62] Schreier J A, Houck A A, Koch J, Schuster D I, Johnson B R, Chow J M, Gambetta J M, Majer J, Frunzio L, Devoret M H, Girvin S M and Schoelkopf R J 2008 *Phys. Rev. B* **77** 180502(R)
- [63] Reed M D, DiCarlo L, Nigg S E, Sun L, Frunzio L, Girvin S M and Schoelkopf R J 2012 *Nature* **482** 382
- [64] Chow J M, Gambetta J M, Magesan E, Abraham D W, Cross A W, Johnson B R, Masluk N A, Ryan C A, Smolin J A, Srinivasan S J and

- Steffen M 2014 *Nat. Commun.* **5** 4015
- [65] Chang J B, Vissers M R, Córcoles A D, Sandberg M, Gao J S, Abraham D W, Chow J M, Gambetta J M, Rothwell M B, Keefe G A, Steffen M and Pappas D P 2013 *Appl. Phys. Lett.* **103** 012602
- [66] Steffen L, Salathe Y, Oppliger M, Kurpiers P, Baur M, Lang C, Eichler C, Hellmann G P, Fedorov A and Wallraff A 2013 *Nature* **500** 319
- [67] Hoi I C, Wilson C M, Johansson G, Palomaki T, Peropadre B and Delsing P 2011 *Phys. Rev. Lett.* **107** 073601
- [68] Peropadre B, Zueco D, Wulschner F, Deppe F, Marx A, Gross R and Ripoll J J G 2013 *Phys. Rev. B* **87** 134504
- [69] Hu Y and Tian L 2011 *Phys. Rev. Lett.* **106** 257002
- [70] Yin Y, Chen Y, Sank D, O'Malley P J J, White T C, Barends R, Kelly J, Lucero E, Mariantoni M, Megrant A, Neill C, Vainsencher A, Wenner J, Korotkov A N, Cleland A N and Martinis J M 2013 *Phys. Rev. Lett.* **110** 107001
- [71] Strand J D, Ware M, Beaudoin F, Ohki T A, Johnson B R, Blais A and Plourde B L T 2013 *Phys. Rev. B* **87** 220505(R)
- [72] Göppl M, Fragner A, Baur M, Bianchetti R, Filipp S, Fink J M, Leek P J, Puebla G, Steffen L and Wallraff A 2008 *J. Appl. Phys.* **104** 113904
- [73] Nakamura Y, Pashkin Y and Tsai J S 1999 *Nature* **398** 786
- [74] Shnirman A, Schön G and Hermon Z 1997 *Phys. Rev. Lett.* **79** 2371
- [75] Martinis J M 2009 *Quantum Inf. Proc.* **8** 81